Toward a Conception of the Holistic

Part 3 of 4

Franklin Merrell-Wolff September 21, 1952

Last Sunday, we covered a preliminary ground in which we became, presumptively, more familiar with the meaning of the word 'finite'; and we perhaps all had some experience of an enlargement of our previously existing ideas as to how big finite can be. As a matter of fact we did not deal with any conceptions that were really difficult at all. The difficulty was in the domain of trying to expand the perceptual imagination to grasp notions which conceptually are rather simple. One lesson that should have come out of that experience is this: that the perceptual power is very definitely restricted. In what we shall do tonight, we will have to drop the perceptual power and operate with other cognitive powers.

Preliminary to what we shall say tonight, I shall outline three cognitive facets or powers: first of all perception, which we shall understand as the cognitive aspect of sensuous experience. The impressions we get from the world get organized more or less automatically into what we call percepts which are characterized by these qualities: that they are concrete and particular, but they are also definitely finite in their limitations. Last Sunday we sought to expand perceptual imagination so as to grasp something of the meaning of a googolplex, or that is $10^{10^{100}}$ —a pretty big number.

The second cognitive power we will call conception. It is a cognitive power that is non-sensuous in its purity; however much in common usage it may be more or less confusedly blended with perception, in its purity and in its most efficient operation, it achieves a high degree of freedom from the restrictions of the perceptual consciousness. It is characterized by generality, impersonality, and definitiveness. While these features are present in variable degrees as among different concepts, in their highest development we get an extreme generality, and an extreme definitiveness, and it's on that level that mathematics exists.

The third form of cognition is one that is practically without recognition in the vast bulk of Western philosophy and psychology, but not totally without recognition. There are at least references among the German Idealists that point to it. By "introception," I mean a cognitive power which transcends the subject-object relationship; but like perception, its content, if you may use that term, is concrete; but like unto conception, its content is completely universal, not particular. Its key word is 'light'. You might call it cognition as pure light. In its purity it operates only in the domain of the infinite. It can be realized, and when realized in its purity, the sensuous or perceptual world drops away, vanishes, and, likewise, the conceptual world drops away and vanishes.

There are possibilities of an interblending between these three cognitive components. In our work last Sunday, we dealt with an interblending between perception and conception; in other words, a domain that's familiar more or less to everyone.

Tonight, as far as may be, we will attempt to drop the perceptual component, since its too weak to journey on into the domains in which we propose to enter, and we'll see if we cannot in some measure effect a fusion of the introceptual with the conceptual. I may say this about the vast majority of mathematicians, that they operate on the level of the conceptual, highly freed from the perceptual, but without the light of the introceptual. When you have the fusion of the introceptual and the conceptual, you have a different domain from that which is familiar to most mathematicians. You have spontaneous luminousness combined with the principle of organization.

Now, we have before us a far more difficult task than that of trying to comprehend the googolplex. Let us consider the totality of all natural numbers. Those consist of simply the positive integers, the 1, 2, 3, 4, and so on beyond all limits. One number and only one in that series is the googol, and another one is the googolplex. Consider this whole series as one entity. That means consider all possible integers whatsoever, and remember there is no such thing as a last integer. Embrace that totality as one entity. Now, you cannot embrace it in the sense of putting a circle around it. You could in principle put a circle around the googolplex. Your embracing has to be done in another way. Let us say we symbolize it by the arms held out this way with an open space, not making a closed circle. The arms defining a zone in one sense. The open space indicating a limitlessness. But the task brought to bear upon the conceptual imagination now is to grasp that totality as just one entity.

We'll have to go further than that. We're indebted to two German mathematicians of the last century for the definite defining and characterization of the infinite. These two are Dedekind and Georg Cantor. It is noteworthy in the work of Dedekind, in his essay on the nature and meaning of number, that you hardly ever see in that essay our ordinary numbers at all. It is an essay about sets and classes—about the primary ideas in the mind; and theorem after theorem developing with that simple material derives the most fundamental properties of number. Some of these we spoke of last Sunday-that number grows out of the establishing of a one-to-one correlation between two classes. We took it back to the stage of the infant and of the primitive. We saw how correlation probably first was made with the fingers of the hand and various objects, later with pebbles and various classes of objects like sheep, and so on. That was before notions of number as we have them were born. That's fundamental counting. That is fundamental number. The basic notion upon which we build is that we can call two classes similar, or in ordinary language, equal when we can set up a one-to-one correlation between the two classes so that there are none left over in either class. Thus if there were five coins and five pebbles, we could set up, even if we didn't know the word five or the number 5-that notion hadn't been born-we could draw a line between a pebble and a coin, pebble and a coin, and exhaust the two classes at the same moment. When that happens we say they have the same cardinality—the cardinal number, which is the quantity number rather than the order number. Better get used to the word cardinality because we're dealing with notions that are very fundamental.

And just as a interjection at this point: I may next Sunday, or sometime later, deal with some preliminary efforts along the line of what we might call a construction of an holistic mathematic, just some preliminary ideas, but to achieve any understanding of even the initial ideas, you have to grasp the conceptions with which we are dealing tonight. The reasons for that will later appear. But now we're going to note the property that's peculiar of our class of numbers.

I'll put down 1, 2, 3, 4, 5 ... —a dotted line afterwards which means it goes on forever. And I'm going to put another line below which will be the doubling of each of the first numbers. Now, here's a very important point: we can set up a one-to-one correlation between these to classes or sets-put this line here; that's counting them. If there are just as many in the one set as there are in the other then they have the same cardinality. Now, isn't it evident to you that no matter how far we go in the first set, we'll always have a number corresponding to each number in that set in the second set? Corresponding to any number *n* over here, there'll be a 2n here. There'll always be a 2ncorresponding to the *n*. Therefore there are just as many numbers, just as many elements, in the second set as there are in the first set. But another important fact, every element in the second set is to be found in the first set: 2 is found over here, 4 over here, 6 over in there, and so on. Yet there are elements in the first set that are not found in the second set; 1 is not found in the second, 3 is not, in fact every odd number is not found in it. There are just as many in one set as in the other. The totality of elements in the second set is the same as the totality of elements in the first set. They have the same cardinality. But the second set is a subset of this because all of it is found in that, but not all of that is found in this.

Now that quality, that property, is the definition of an infinite class. An infinite class is a class which has one or more parts, proper parts, which have the same cardinality, that's the same number or totality, as the whole; a proper part which has the same cardinality as that of the whole. That is never true of any finite collection or a finite class. If you take a proper part of a googol, for instance, if you take 100, a subset of 100, out of that googol, the googol will be reduced by that one hundred. You can't set up a one-to-one correlation. It does not have a proper part which has as many elements in it as the whole. Only infinite classes or sets have this property.

Now, there's some very wonderful things you can do with our integers. Would you believe that you could count with the integers all the rational fractions? Just consider the rational fractions between 0 and 1. It's obvious isn't it, at once, that there's an infinity of them in there: $1/10^{10}$ would be one of the fractions in there, 1/2, 1/3, all the fractions with 1 in the numerator and any number in the denominator, and several with a larger number than 1 in the numerator, and that between 1 and 2 you find a similar infinity, and so on between every integer whatsoever out through the whole series. You would have an infinity of fractions between every one. Is that clear? Any one to whom it is not clear?

Now what we propose to do is to count the sum total of all fractions in the number system that goes out to infinity. What do we have to do to do that? We have to order them in a definitive way such that we'll be sure of picking up every fraction whatsoever. Think about it. How would you go about that? How would you start out a system that would enable you to know certainly that in that system you had all of your numbers, rational numbers, fractions and integers, so ordered that you had them all. Now you couldn't start out from zero and say you take the next fraction. It wouldn't be 1/2; it wouldn't be 1/1010. There's an infinity of them between a googolplex and zero. Now, we don't want the problem of trying to order them so that we can start counting. You can't count until you can order. It so happens that this was worked out in a very clever

way and a rather simple way. Let us write all the numbers in this fashion. Now, let's see we'll, and just. We'll write every number as a fraction. We'll start in here with 1/1, the numerator will always be 1s on this line, but the denominator will correspond to the number there: 1/3, 1/4, on to infinity; and down here we'll have 1/2, no wait a minute, no, no, 2/1, 2/2, 2/3, 2/4; and we have 3/1, 3/2, 3/3, 3/4; down here we have 4/1, 4/2, 4/3, 4/4, and so on. Now, if you follow that system out clear to infinity in every direction, you will get every fraction that there is in an orderly arrangement including every whole number, for the 1/1 reduces to a whole number, 2/2 reduces to the same one, 3/1 reduces to 3. You'll have every integer, positive integer; you'll have every fraction whatsoever somewhere in that system, and they're nicely ordered. This will show right here. See these, all of these continue on to infinity—this way and that way.

Now, suppose we start and instead of writing it that way we write them (1,1) that stands for that fraction. It's just a different way of writing it. Then from here we go over here; we write (1,2), and then we come down here on a diagonal—pick that up—we get (2,1), then we go back over here and we get (1,3), we get (2,2), and (3,1), and so on. Then we can set up our one-to-one correlation— 1, tied in here; this is the second one. This is the counting process now. This series—you can write this very simply now if vou'll notice certain rules. The sum of the two keeps ascending. There'll be some with the same sum—that is, in here 1 + 2 = 3 and 2 + 1 = 3. You write them in the order that the ascending numerators. Hence you have perfect order then. This numerator is low; this numerator is larger. Then you take the ascending numerators here. The first number being the numerator in each case; the second number being the denominator of the fraction. Now, do you see that here you will actually pick up every rational fraction there is, that you've given to it a definite order, and that now you can count it? And no matter how far you go out here you'll always have a whole number, an integer, that will correspond to your rational fraction. Hence, there are enough integers to count not only all the integers, but all the integers and all of the rational fractions in addition.

This is the mathematics of the infinite now, not the mathematics of finite quantities. It's a different dimension of mental process, but it just so happens that this correlates and gives a rational pattern to many reports from mystical experience. Experiences that appear to the ordinary consciousness as quite irrational when they're formulated, when you use this kind of logic they fall into a comprehensible and rational form. That's where the importance of this comes in. In fact you don't have to say thinking has to stop when you get over into at least some dimensions of the transcendent. We're dealing with an instrument that enables us to carry a kind of thinking over. That's where the importance of following this kind of reasoning comes in.

Now, what we're using here in our one-to-one correlation is precisely what primitive man did when he counted with his fingers. And if you are justified in saying that if you get a correspondence with these five fingers among certain objects, if you're justified in saying that they have the same cardinality, using the same process, exactly the same process, just as rigidly, we can say of this series here that it has the same cardinality as that. The series or collection consisting of all whole numbers plus rational fractions can be counted by all of the whole numbers. You've got to forget all the rules held in your ordinary grammar school arithmetic. This is another domain.

Now, this infinite, an infinite like this that can be counted is called a denumerable infinite. The idea is that if you could count for infinite time you could count them all. Later we will have to consider the infinites that cannot be counted. Now we have another thing. This is a simple one. This is not severe yet. The next one we won't attempt. I'll just point out the fact that a further proof was made that not only the whole of rational numbers but the whole of algebraic numbers can be counted. Algebraic numbers include all rational numbers plus a large number of irrationals like the $\sqrt{2}$, and imaginaries like the $\sqrt{3}$, or complex numbers like $a^{\sqrt{3}} + b$. They are numbers, the technical definition of which you probably wouldn't understand and wouldn't be expected to understand, but they are the numbers that can be the solution of algebraic equations of any degree having integral coefficients. The class of numbers is so large that we ordinarily represent them by a two dimensional plane. We put our ordinary numbers which we call commonly the real numbers—just a name—and so on; now we have the minus numbers in the opposite direction; and we have on this vertical line the imaginary numbers—and we have $-\sqrt{-1}$, -2x $\sqrt{-1}$, $-3 \ge \sqrt{-1}$, so on indefinitely in each case. And out here we would have points in space; this number would consist of 3—this one underneath, like that— +2 into $\sqrt{-1}$; that's called a complex number. And this space would be filled all full of every possible number with every possible fraction whatsoever. We take that up as a convenient way of representing them. Further proof was made by Cantor that the sum total of all of these numbers that can be solutions of algebraic equations can be counted by the rational integers. We won't try to do anything more than just state that fact.

But we come now to the next step—a proof, although there's at this point some differences of opinion, that you cannot count the total of all real numbers. Real numbers consist of those that are not imaginary—like our integers, like our fractions, a/b, all of these simple irrationals, cube root of 7, and so on. All of these that I put on the board here can be solutions of algebraic equations with integral coefficients, but the real numbers include numbers like π , the ratio of the diameter to the circumference of a circle, and e the number is written this way—one of the simple ways of—the limit when *n* equals infinity of $(1 + \frac{1}{n})^n$ —two numbers of enormous importance: π you can appreciate; e, the base of our natural system of logarithms, but more important than that in one respect, it is found that wherever you study the phenomena of life, get the statistical data connected with anything that is living, draw your curves that correspond to your statistical data giving your life cycle, the curve when reduced to a formula or to an expression always involves the number e. There's some mystery in this, but e is the number of life. Now, e and π are transcendental numbers. That means technically that they cannot be a solution of algebraic equations having integral coefficients. At the time of Cantor, these were the only transcendentals definitely known. They're an awfully hard number to find, but his proof was that they are so much more numerous that when added to the other numbers they cannot be counted.

Now, here's a little bit of proof that begins to take you into the domain of higher mathematics—the reasoning that's sometimes used in higher mathematics. Facing the problem of ordering all the real numbers—that doesn't mean merely the rational numbers, integers and fractions, which it was easy to order; but now you're going to try to order all the real numbers. That means you'd have to get in every irrational and every transcendental there would be. It's an impossible order; there's no way of doing it, at

least that a human mind can envisage. Now, Cantor suspected that the number of real numbers was so great that you could not count them even with an infinity of integers.

Now let us consider the region from 0 to 1. If we could prove that you could not count all of the real numbers between 0 and 1, then obviously you couldn't count all of the real numbers from 0 to infinity. So all we have to consider if we're going to prove they can't be counted is the region from 0 to 1. Let us take every fraction—these would all be fractions—write them as nonterminating decimals. Thus, for instance, some are naturally nonterminating, most of them would be, and if you had a .4, which is complete; in the nonterminating form you write it this way: 3.399..., and your nines go on to infinity, and when you get to infinity this number is just as big as the .4. So you can write everyone of those terminating decimals or fractions in a nonterminating form. Now we're going to write all of the numbers between 0 and 1 in a nonterminating form. See if you can follow this. Yet we can't find an order, so we assume that an order exists. We'll write our first number, and let us use letters to represent the digits in our fractions. We have a_1 $a_2 a_3$, and so on to infinity. Now our next one, we're just saying arbitrarily that an ordered way has been found; maybe only that God himself could find that order, but we just say it exists-that it exists somehow and we let these letters stand for it. This is where the reasoning gets subtle. Go on, we have $c_1 b_1 b_2$ no, c_1 and c_2 , so on; that would be. Now this goes on down here to infinity too-infinity in both directions. Now, assuming that we've been able to order them and count them we examine it to see if there's any difficulty, and at once a difficulty arises because we can find that there's an infinity of numbers no matter how we arrange this that will not be included. For instance, consider the numbers that you get by taking that diagonal down. Write another number where you change this digit in the series, and change that digit, change that digit, and so on clear through; you'll clearly have a number that will be different at least in one place from any number that you may have in this series. Can you see that? That means, then, that you have not got all of the numbers in. As a matter of fact, since you could do that on any of these diagonals there would be an infinity of numbers that you could not have included no matter how far or completely you wrote this out. In other words, our assumption that you could count them, order them and count them was proven false. The other conclusion is that they are nondenumerable; that they are so numerous that the infinity of digits that could count all our fractions, all our algebraic numbers, and could count all of those numbers though you multiplied them by infinity-still could count them-could not count the sum total of all real numbers.

Now here is where the logic gets subtle. The principle that's employed is this: first we say that the totality of all real numbers is countable or it is not countable. If you find that when you assume that it is countable you run into a contradiction, then the conclusion must be that it's non-countable. That's the dichotomy. The question of whether this reasoning is sound or not depends upon whether the dichotomy is valid. Now it is, for instance, if you where to say that an equation is either reducible or not reducible, you would have two classes. It belongs one way or the other. Does that hold? Is there some middle ground which belongs to the zone of that which is "not reducible" and "not not reducible"? Some criticism of the reasoning here has been brought from that angle. But if we accept the soundness of the *reductio ad absurdum*, then it follows that the sum total of all real numbers is more than a denumerable infinite.

Now here's the interesting fact. At Cantor's time two transfinite numbers were known. Since then several classes of an infinite number of transfinite numbers have been discovered. They are immeasurably, infinitely more numerous than all the other numbers put together, and yet they are hard to discover and only two of them are well-known to everybody, namely π and e. Let us suppose we took all other numbers than these transcendentals—all the algebraic numbers, integers, rational fractions, and the ordinary irrationals, and the imaginaries, and complex numbers—and we placed them out in space as I showed before. We'd find this true: that between any two of those numbers that would correspond to specific points, like the points I have listed here which we'll say are $a^{\sqrt{3}} + b$ and $c^{\sqrt{3}} + d$. Between any two points you can always find another number. You can always find another number. Do you see that from that statement it follows that you can always find an infinity of other numbers? Now here's a check of your logical sense. If between a and b, or 1 and 2, I can find another number, if it always is true that between any two numbers I can find another number, then it must follow that between those two numbers I can find an infinity of other numbers-of course quite obviously. Between your a and your b another number, we'll call that c, but our rule would say that somewheres between c and b we'd find another number, and so on ad infinitum.

There's another feature of mathematical thinking that's very fundamental. It's part of the step from "any-ness" to "every-ness." If you can say something about any member of a class, or set, or group, or collection, by "any" we mean one picked at random, whatever we can say of any, we can say of every. See, we're picking out any on the basis of its general property, not about particularities that may attach to special entities.

Now it would seem, would it not, to you that after we got down all of the algebraic numbers, all of these numbers we've been talking about except transcendentals, that that plane would be pretty solidly filled. wouldn't it? Remembering that you can always get an infinity of numbers between any two points; yet as a matter of fact, that would be like a sky with the numbers corresponding to points like stars with vast black, blank spaces in between. Your plane is not tightly filled. Remember, your points have no area at all. They're absolutely sharp—*arealess*. They haven't packed that plane; but actually that plane has infinitely greater spaces in it than the space that would correspond to the numbers. In other words, without the transcendentals you don't have a true continuum. The only way you can fill that space is by bringing all of the transcendentals in. And I think you can begin to see the enormous vastness that belongs to the transcendental numbers as compared to all of the other numbers.

So, one simple notion of infinity is not enough to take care of our total problem of determining the cardinality of all possible classes. This leads us to what you might call a hierarchy of infinities. The first infinity that corresponds to the total of all integers, which was sufficient to count integers and fractions, and in addition sufficient to count all algebraic numbers, has been written variously as Aleph-Null, or \aleph_0 , it sometimes has been written Ω_0 , one is the first letter of the Hebrew alphabet and the other is about the last letter of the Greek. And this is known as the denumerable infinite corresponding to the cardinality of all integers. The cardinality of all real numbers is more than infinitely greater than that. And we have a very interesting multiplication table. There are certain laws that attach to these numbers. And we take \aleph_0 add 1 to it and the answer is just \aleph_0 ;

add a googolplex to it and it swallows it just as easily. The answer is \aleph_0 . Just swallows it as easily as it does the 1. Or, again, if we subtracted a googolplex from it, have \aleph_0 minus a googolplex, which is $10^{10^{100}}$, and that just equals \aleph_0 . You can't disturb its calm in that way. You see a whole universe like this could drop out and it would just go on just as placidly as you please, nothing happens. Or, now, let us try something else, see what multiplying would do. If you multiply it by a googolplex, that is $10^{10^{100}}$ times—I can't justify these statements; you just have to take them; I think that's enough anyhow what'll happen? It swallows it up just as easily. It hasn't changed it at all. The multiply it by itself, that's equal to $(\aleph_0)^2$, it just keeps \aleph_0 . You haven't disturbed it yet. It takes something more powerful to disturb it than that. That means none of these processes have taken us out of the domain of the denumerable infinite. This is what you have to do to have any affect on that number. You take $(\aleph_0)^{\aleph_0}$ and that'll give you—at last it does something—you get \aleph_1 ; the second transfinite number.

Now you say do these have any correspondence? Does this number correspond to anything? It corresponds to the cardinality of the totality of all real numbers including the transcendentals and the cardinality of the continuum. That is the mass of numbers it would take to make all of this space solid. And the same effect of multiplication and addition applies to \aleph_1 . As a matter of fact $(\aleph_1)^{\aleph_0}$ just swallows that up and it remains \aleph_1 . It's unaffected by it. The only thing that affects it is raising it to the \aleph_1 power, in which case it achieves a higher cardinality and becomes \aleph_2 . Now, there's some evidence that this corresponds to the number of single valued functions, but you won't understand that, and those, so far as I know, are the only ones that have usage.

Now let us assume the process carried to the limit and we get this: \aleph_{∞} —the symbol of the whole, the Holistic, the most comprehensive conception. It evolved in the mind of man, and since the mind of man is a part of the whole, it could not evolve something greater than the whole; hence, as the most adequate symbol of the vastitude of the whole: \aleph_{∞} . Your googolplex by now is a microscopic pellet. In the sea of the illimitable, the whole galactic universe, nay a denumerable infinity of galactic universes of the same size would dissolve into a submicroscopic insignificance.

It really makes no difference whether you call the universe an illusion, as Shankara and as the Buddhists do, or whether you call it real, as Sri Aurobindo does; in any case, in the presence of the multiple infinitude of the whole, they are absorbed as insignificant irrelevancies. Hence, whether real or illusion is not a point of vital importance.

When a mathematician speaks of the infinite, he does not mean merely a big number. He means things like this series of which we have spoken. But he means in differentiating between infinities of different orders, that they still have a character; that it is not a blank of largeness in which there is no element of determinateness at all, but rather that they have a character so that there is something distinguishable—a hierarchy of infinities.

Now the question would arise: how could a finite creature ever know, ever realize the infinite. The answer is a finite creature never could, for the finite creature would be limited to a progression of finite steps, and in a finite time could never realize the infinite. But if the reality of man, nay the reality of all creatures, of all entities, is that they are part and parcel of the infinite, not merely cutoff apparent finite fractions, but coextensive with the infinite, then the infinite is knowable in the sense of Realization by the simple removal of an obscuration; the seeming of finitude being placed by some instrument of obscuration.

I considered it very significant when Dedekind gave his existence theorem concerning the reality or existence of infinite manifolds: he said take the ideas in the human mind; you can have an idea, which we'll call a_1 , and then you can have an idea of that idea, which we'll call a', and then the a' can be put in the first series as an object of thought, and your a_2 would be the idea of this idea, and that can be placed up there, and the process continues indefinitely. Every idea in the second series would be in the first, but there'd be one idea in the first series that was not in the second. Particularly, he gives the idea of our own ego as one that's not included in the second. You have a cardinality—equal cardinality, the same cardinality because of the one-to-one relationship. Therefore, the ideas in the human mind are infinite. Now, that doesn't mean that they're infinite in the sense of actual concrete thinking of an infinity of ideas, you might say infinite by this power of a generating progression; that the very power to generate the progression and to see it points to its infinitude.

I know these ideas have some subtleties in them. They're too easily grasped. I'm quite sure that the lecture of last Sunday probably seems rather simple now and that the googolplex is something you may take in your stride relative to this. I've been thinking during the last few days of a possibility of formulating the first principle for what we might call a holistic mathematics; and I might by next Sunday be prepared to give a first talk on this, but I'll have to assume that you're familiar with the kind of thinking we've been doing tonight. It'll prove necessary if we are going to use the basic holistic conceptions and use the mathematics of the transfinite. So this is preparatory, in one sense, to that as the other part of its meaning was to give some more adequate understanding of what is meant when we speak of the whole. No simple denumerable infinite, but a vast nondenumerable infinity compounded an infinity of times. Naturally, we sink as relative beings into a less than microscopic significance compared to that, but he who knows that this vast, this which is none other than *Parabrahm*, is that with which in truth I am identical, need not identify himself with an insignificant finite appearance, but may know, as Shankara said, that he is not only a part of *Parabrahm*, but that he's identical with the whole of Parabrahm. Now let us add to that Sri Aurobindo's insistence upon the persistence of individuality. By the use of the conceptions we employed tonight, it is quite readily possible to reconcile those two statements of identity with the whole of Parabrahm, the whole of the holistic, and yet infinite variety of individuality.

That, I think, is enough for tonight.