Mathematics, Philosophy, and Yoga

Part 2 of 6

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Good evening everybody. There were a few questions, and we'll deal with those first. One referring to *The Song of Life,* with which I am not familiar, I will not handle.

"What is *Raja* yoga?" Well, it's a technical form of yoga, akin to but different from *Hatha* yoga, dealing primarily with certain psychic processes and to some extent with physical processes. It's more to be regarded as in the nature of an aid rather than being one of the fundamental *Trimarga*, which are the routes of knowledge, love, and action or will.

"Is there such a thing as Integral yoga?" Well, I believe so. That of course is, as probably all of you know, a term particularly associated with the name of Sri Aurobindo. The idea being that it is not sufficient or quite comprehensive to reach the goal by one way alone, but to follow the three-fold path of the *Trimarga* either simultaneously or in succession. How successful that has been, I do not know. There's nothing certainly wrong with the idea.

Now, "Is Aurobindo what you would consider enlightened?" Well, I would. It is just an accident that I didn't mention him last night. I so regard him. I would not attempt to measure his stature or that of any other illuminati; for after all, when the tops of mountains are above the clouds, it's useless to try to determine how high the peaks are relatively. Usually when one says that such and such a being is the greatest of them all, and of course it would be the one who is particularly close to him, he is reflecting ego—my God, my teacher, my guru—emphasizing that, and this is the greatest of all, is in the nature of a form of subtle egoism. Do not attempt to measure the height of those whose transcend earthly vision. It is enough that any one of them can illumine the way. That covers that.

There's one other question here. "What is there which is not either *a* or *not-a*?" The whole of Enlightenment, and that's the simple answer. This denial that that which you seek is either the *a* or the *not-a* means that the dualistic consciousness cannot comprehend it; that it falls outside those limitations. Our universes of discourse, that's still on the board, are valid only within the limits of the dualistic consciousness. That's the point there. Now, you say, "I cannot imagine another consciousness, something else." Yes, that's perfectly true. How could the limited dualistic consciousness imagine the nondualistic? The road there is by Realization, and after Realization you know; before Realization, you do not know. Do not try, therefore, to define that which lies beyond. Definition applies validly within the realm of dualistic consciousness, but our concepts are not made for that which lies beyond, and only by artful use of them can they even suggest something of that.

Now, he goes on to continue his question, “Is not even matter fluidic?” Now this is a question that calls for many counter questions, and this could take us into deep waters. What do you mean by “matter”? Do you mean a convenient hypothesis for a concept uniting the impressions that come to you through the senses? Or are you hypostasizing this that you know through the senses—touch, sight, sound, pressure, kinesthetic tension. Those are the things I know. Are you hypostasizing that and mean by matter something that exists externally beyond all consciousness in every sense—a nonconscious somewhat? If you do, how do you know it? What you know are these impressions. That’s all. Now, I did not yesterday mention the word matter. I mentioned two functions, or organs, or faculties of cognition, namely, sense perception and conceptual cognition. We are familiar with these contents; that we know. But do they give us messages of a somewhat which lies beyond all consciousness? Oh, it’s our habit to so regard them, but that’s careless thinking; and when you come to reading the Buddhist *sutras*, if you do, you’ll find that they’re very exacting on that. You’ll find the Buddha at least represented as saying the qualities are all we have—and by the qualities he means that which you actually experience—and there is no more. When you speak of a substance that is not itself experienced in which those qualities inhere, you’re not rigorous. That you do not know. It’s a careless habit.

This question could take us into deep and rough waters, and I have intended to deal with the problem involved here later after there’s been more preparation, for you’re dealing with a question that has occupied philosophers for many years, and as a matter of fact, the fully informed modern scientist makes no metaphysical projection of an idea of matter. I’m speaking of the really intelligent scientists, not the mere scientific clerks that work on problems—I mean men like Einstein, Vannevar Bush, for instance—know that that which they deal with is only a certain set of determinations largely capable of only mathematical formulation. From their experience, they project certain hypotheses that seem to integrate these experiences in a conceptual whole. Those hypotheses are good if they lead to further experiments or further observations that are predicated by the hypotheses, and so confirmed. They fail if they do not so confirm. A theoretical physicist that I knew said, what is obviously true to any mathematician, that there’s a potential infinity of possible explanations of all the phenomena that we have, and I can illustrate the point again from a mathematical reference.

One of the simplest examples in the history of science, one of the most beautiful, was Kepler’s determination from the observations of Tycho Brahe that the planets in their paths around the sun followed fairly closely the path of an ellipse. Now, this could be a paradigm for all building of hypotheses. You determine certain points. Let those points represent observations—classified observations. Here we’ll put down five of them. In his case, being a spatial determination, that was pretty precisely what he did. Now, if you know your conic sections, or the equations of the second degree, which consist of a circle, ellipse, parabola, hyperbola, and two intersecting lines, it is a fact that any five points determine uniquely any conic section. And in his case, these observations, along with others, did so determine a path—not as extreme as that. Of course, with the planets it is not an ellipse of such eccentricity, they’re almost more nearly circular, but that illustrates the point. Let your equation represent your connecting hypothesis, your postulated interpretation. If you set the restriction that it must be a curve of the second degree, five points will give you a unique determination; but why should you set that restriction with respect to your observations—changing our figure now to any observations in a scientific problem? Might it not be that the true explanation would be in the form of a higher plane curve—higher than second degree? Those higher than second degree, those of third, fourth, fifth, on to the *n*th degree, are infinitely numerous, and there would be literally an infinity of curves that would pass through those points representing observations; and, thus, theoretically it is possible to have a hypothetical explanation or a theory explain the facts of scientific determination.

The scientists also impose a certain arbitrary rule upon hypotheses that is not required by the necessities of logic itself, and that is that the hypothesis shall be of such a nature that it shall lead to suggested experiments or observations for further checking. It's known as the "operational requirement." But have we any certainty that ultimate truth is of such a nature that we can check it by our methods within the dualistic consciousness? What we do have in science is a pragmatic test of truth and not a test of truth *per se*. And by pragmatic I simply mean that it works—that these hypotheses lead to experiences that can be predicted, that if you put your foot down on the starter in the automobile, the motor will turn over. Something you predicted came true. In a practical pragmatic sense, you've checked it. That's all. But the truth that works within the realm of dualistic consciousness is by no means necessarily the true truth, the truth as it is in itself.

Now, a man like Einstein is fully aware of this and has said so. And one of the men that he most admired was Sir Isaac Newton, the man whose philosophic concept of the nature of the cosmos, he overthrew. He is a step beyond Newton in that Newton still believed in metaphysical existences and introduced them into his hypothesis. He had a conception of an absolute time moving through space evenly, which on examination has no meaning practically. Many of his laws work. They still work for our experiences within the megascopic range, but they are no longer valid, and they know them to be no longer valid, when we deal with velocities approaching that of light or with determinations on a great cosmic scale, determinations far vaster than were possible in Newton's time. So Einstein said what Newton did was give the first approximation to the truth, and what I've added in the light of a greater experience in the field of science is only a second approximation, and others may follow later with other changes. He had the conception that here was an infinite series of steps of which these were the first two approaching truth. There you have the modesty of the really great scientific mind, one who knows the limitations of his own methods.

One of the most significant things in the step from Newton to Einstein is that now epistemology enters into the picture. The ways that we have of determining fact, like a light signal from a star, condition the form of the knowledge. The knowledge is thus relative to the limitations of our knowing process. That's a vast maturity over the earlier integration. It was not possible back in the time of Newton, for we had not yet awakened to the limitations of the knowing process with which we deal as we have since. We're locked within this dualistic system, within the limits of a limited means of cognition. As I said, two arms of cognition: sense perception and conceptual cognition—cognition that moves in terms of concepts having certain character.

And now that was all I talked about last night and I never mentioned matter. And when I spoke of the realm below the line that I put upon the board, and I identified it with granular consciousness or with the *manifold* of individual elements each of which could be next to the other like the number system, considering only the natural numbers. I used the concept of the *continuum* to represent that which is above the line—beyond dualistic consciousness. I made only an approximation—only an approximation to the truth—for after all, this continuum is granular also; again, marked by the limitations of our cognitive process. But our granules get pretty small. They actually become infinitely small and are called "infinitesimals." At least that is the concept left by Leibniz, and at least in my day that was the form that the calculus took. We dealt with a notion of infinitely small grains, literally infinitely small, that make up the parts of the continuum. But since they are grains, it is not pure flow.

Now, the mathematicians haven't ever been content or really satisfied with the idea of something, an existent that is infinitely small—small beyond all measure; and it is said that Weierstrass disposed of the infinitesimal for all time. He is rated as the greatest of the analytic thinkers. A man that you might be interested in because he looks like a poet and this remark has been attributed to him: "The mathematician who is not also something of a poet is not a complete mathematician." Now, you get perhaps here a little of the feel of the mathematician. I'll give you a little more of it as we go on. He got rid, so they say, of the infinitesimal, but at a price—the price is the denial that there is any such thing as motion, that all we have is bodies occupying points in space stationary corresponding to points in time. Now, it works. You can build a calculus on that basis alright. I have not seen the analysis. It was not taught in our calculus in my day. We followed the line of Leibniz in which you tend to regard infinitely small entities as in some sense real entities. Now, you can get rid of those infinitesimals at the price of abandoning the notion that there is any such thing as motion, but only bodies occupying different points in space corresponding to different points in time. Your intuition might say, well how could they get to the different point without moving there. It's found it's unnecessary to make any use of that intuition. All you need to do is to consider motion as merely—that's what we call motion—as merely matter stationarily, or bodies stationarily occupying different points in space at different points in time. You might call it the cinematographic view of reality; and that the idea of motion is simply an illusion or *maya*. Now, I'm not championing this point of view. I'm merely presenting it. I'm standing neutral at present. Now, we do know it's possible to use a series of stationary images that'll produce an illusion of motion. Every time you go to a moving picture that happens. Every image that is thrown upon the screen is completely stationary, merely they're thrown on very rapidly and you get a sense of flow or motion, but there is none there.

Again, there was a Greek philosopher by the name of Parmenides who long ago maintained there was no such thing as motion. His counter number was Heraclitus who said there's so much motion you can't step into the same stream twice, nor even, after all, once. And Zeno, who was said to be a disciple of Parmenides, developed his paradoxes for the very purpose of showing that if you conceived of such a thing as motion you got into trouble. Thus, the famous paradox of the Archimedes and the turtle: you give a turtle a head start; now, he says, Archimedes could never catch that turtle no matter how slowly the turtle goes or how fast Archimedes travels. For, say Archimedes—or not Archimedes, Achilles—starts at this point and the turtle is over at this point, Achilles has to get to this point before he can catch the turtle, but meanwhile the turtle has gone on, and he has to get to that point before he can catch him, the turtle has meanwhile gone on, and that goes on indefinitely; and there would be an infinite number of steps, and he couldn't possibly consummate an infinite number of steps in finite time. Therefore, there is no motion. Now, you may laugh at it, but logicians and mathematicians have wrestled with that problem for 2000 years, and they're not completely satisfied with the resolution of it yet. There are some real difficulties in this thinking process of ours. Bertrand Russell thought it could be resolved by the idea that you could pass over an infinite number of steps by a finite time because the sum of an infinite number of steps is not necessarily infinite, but may be finite. That may be the resolution, but they had to wait to our day if it is. So don't laugh at Zeno, he put it in a humorous form, but it was a very critical problem in thought.

Now, tonight I was going to go into the mathematical side of things a bit, but first I wanted to say as to my plan in lectures, I try to unite the systematic with the spontaneous. The systematic is that which is normal to ordinary intellectual organization. It's what you get in the professors lecture right along. It's the sort of thing you can do and know you can do it beforehand. It merely takes work. So, I regard anything that's purely systematic as merely a detail because you can say this I will turn out at such and such a time, and I'll produce it at such and such a time, and you can deliver. But the spontaneous is that which comes from out from beyond the cell, from above that line, and that you cannot, through the powers that lie below, say this I shall deliver. It comes as a bestowal, if it comes at all. And when it comes, all that belongs to the systematic is cast aside and this takes over. To maintain the two in operation is a complex problem. It may mean that you abandon all plans that you have had for a lecture even at the moment of coming in, and that has happened to me. In stepping from the door to the platform, I've dropped everything and spoke on a totally different subject without any plan beforehand, and that means for an hour at a time—organization going on in the very process of delivery.

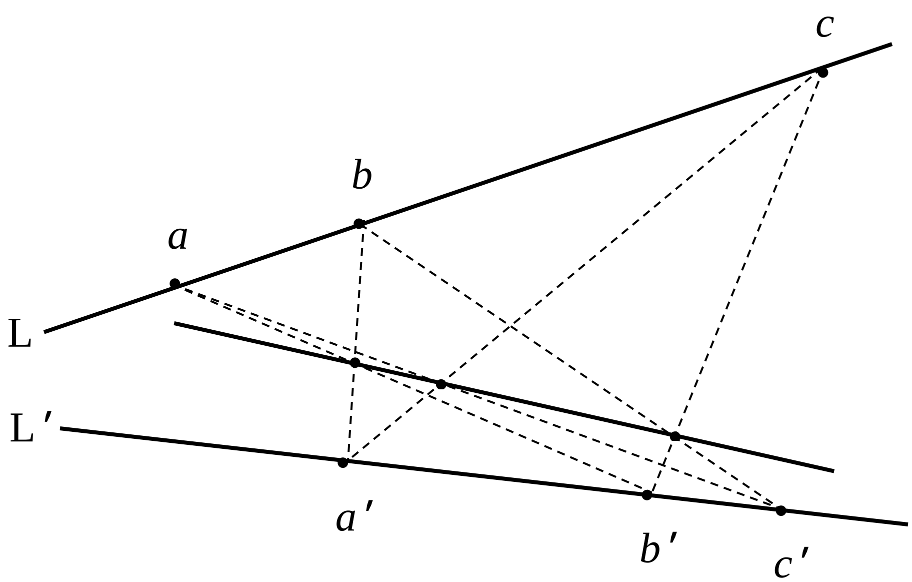
Now, you may predict that such may happen; you cannot say this I shall do. When it does happen, there is a certain phenomenon, and by phenomenon I mean features that are recognizable, detectable, at least by some people. The consciousness beyond the line or veil appears here as a Field Consciousness, and it may be sensed as a something like a deepening, as a palpable silence filled with unformed meaning. It may take over the guidance of conceptual construction, and if it does, it does so with a sure hand and one can do that which in his ordinary capacity he cannot do. It can evoke in those in the vicinity states of the mystic sort, ecstatic states of consciousness—states of delight. It can produce an experience of heat that is not well explained in my mind as yet, but it happens; make no mistake about that. I've seen faces become red, perspiration roll down the faces, and people take their coats off. The effects can be that tangible. And one may speak words that are instructions to himself as well as to others. What is the source of this knowledge? There is more than one; in part, it may the deeper knowledge of that unseen part of one; it may be the knowledge of the Brotherhood—for that Brotherhood is a many in one, not a separated granular structure, but a whole that may be represented in this way: of the "I" becoming "we" and yet remaining "I." And above and beyond this, a knowledge that is not the possession of any self, a knowledge that does not require a knower, but is a sort of universal storehouse, and this is power to mold the consciousness of man, to bring nearer the opening above. So whenever this impinges, it takes primacy over everything else. But it may be here for a time and then may go. I return to the systematic at such times. It is not the private work of one who speaks; it is the joint work of the speaker and the audience. It does not happen in the presence of a hostile audience or of an audience that is all a question mark—what does this guy think he's got, for instance—but it depends upon an inner rapport. It may take time to build that rapport, but it can happen. The states can be very deep, even as deep as waking *samadhi*.

Now, this is a little glimpse of something of the Beyond. The first that it may mean to the relative consciousness as it glimpses into those depths is as though it were a darkness, silence, and emptiness; but shifting to its own level, to another way of cognition, it is known as the intensest light, utter fullness, and the acme of meaning—meaning being the inner essence of sound. Actually, it's here now.

Now, I think I said we'd come to something of definitions. We can shift over. What is mathematics? You know my subject is mathematics, philosophy, and yoga. There is a reason for that combination; it is the path I went, and therefore the one I know best. Now, you look in different dictionaries as I did, we get very many different definitions of what is meant by mathematics. I looked up in the old *Century* and it said mathematics is "the science of quantity."[1] Now, I suppose that's a popular idea, but it's very far from the truth. There's much of mathematics that has nothing to do with quantity; as for instance the algebra of logic, the development of the great Irish mathematician Boole, and concerning whom Bertrand Russell said that he was the first of the pure mathematicians. The algebra of logic is not concerned with quantity. It's concerned with classes, sets, concepts like that, relationships. Another branch that is not quantitative, and incidentally a very beautiful one, is projective geometry. I thought you might be interested in one of the propositions here. In projective geometry there's no metrical property whatsoever, no measurement; and where quantity is considered measurement is fundamental, but these propositions are descriptive. Unfortunately I do not have something good for drawing lines, but now let us draw at random any line, and any other line. We'll call them L and L'. Now at random, we put three points on each line anyplace whatsoever. Now, we call these points a, b, c on L and a', b', and c', on L'. We draw a line between a and b', and a line between a' and b. Note that point there. Draw a line between b and c'.[2] Remember, these points and lines are taken at random, no measurement at all. Now we draw a line from c to a' and from c' to a. And the theorem is—and this is proven—those three points lie on a straight line. To the mathematician that's beautiful, not because of what you see with the eye, but because of its massive unexpectedness. That was proven by Pascal, originally. It's a special case of a larger theorem dealing with the conic sections. But your point is these lines are random. They're infinitely long. All lines in projective geometry are infinitely long because there's no measurement at all involved, there's no consideration of size of angle, your points are taken completely at random, anyplace you want to put 'em on the lines. You draw these and you get the three on one line.

[1] William Dwight Whitney, ed., *The Century Dictionary and Cyclopedia*, vol. 5 (New York: Century Co., 1911), 3659.

[2] Wolff apparently didn't mention the line from b' to c.



Now, there is in that, if you can get it, something of the experience of beauty that appeals to the mathematician. It's a conceptual beauty. It's the very bringing together into a kind of unity of elements that seemed separate, unconnected. There are many such experiences, usually realized only at a high level of concentration, that can produce effects that are definitely ecstatic.

More adequate as a definition of mathematics is the one that is found in Baldwin's *Dictionary of Philosophy and Psychology*, namely, that mathematics is the "science of abstract relationships."[3] Williamson, in his contribution to the ninth edition of the *Encyclopaedia Britannica* says, "Any conception which is completely determined by a finite number of specifications is a mathematical conception."[4] And here's one I do not remember as well by Bertrand Russell, "Pure mathematics is the class of all propositions of the form '*p* implies *q*', where *p* and *q* are propositions containing variables the same in both propositions and no constants except logical constants."[5] I imagine that for many of you the conception of mathematics is somewhat different from what you formerly thought.

Oh, yes, returning to the non-metrical forms of mathematics, besides the algebra of logic and this projective geometry, there is a form that's known as rubber sheet geometry. You study what relationships remain constant if you assume your plane is

[3] James Mark Baldwin, ed., *Dictionary of Philosophy and Psychology*, vol. 2 (New York: Macmillan, 1911), 47.

[4] *The Encyclopedia Britannica*, vol. 15 (Chicago: The Werner Co., 1894), 629:

> Any conception which is definitely and completely determined by means of a finite number of specifications, say by assigning a finite number of elements, is a mathematical conception.

[5] Bertrand Russell and Alfred Whitehead, *Principia Mathematica*, vol. 1 (Cambridge: Cambridge University Press, 1910), 3. Russell's full sentence is: Pure mathematics is the class of all propositions of the form '*p* implies *q*', where *p* and *q* are propositions containing one or more variables, the same in the two propositions, and neither *p* nor *q* contains any constants except logical constants.

being distorted in any way whatsoever. You could distort it in such a way that a square would be made into a circle, or an ellipse, or any other figure. What remains constant? The order of parts remains constant, and that is a subject of considerable importance. Another name for rubber sheet geometry is topology, and there you can have funny experiences like a ring that has only one side. Oh, I wish I had put one together and brought it here. The point being that you can put a pencil and without lifting it make it go around on your sheet of paper running down the center and without lifting it, you'll find it'll go clear around and runs into itself; so it's a ring with only one side. Now, these are some of the queer things that men do investigate, and some of these are far from being a matter of quantity.

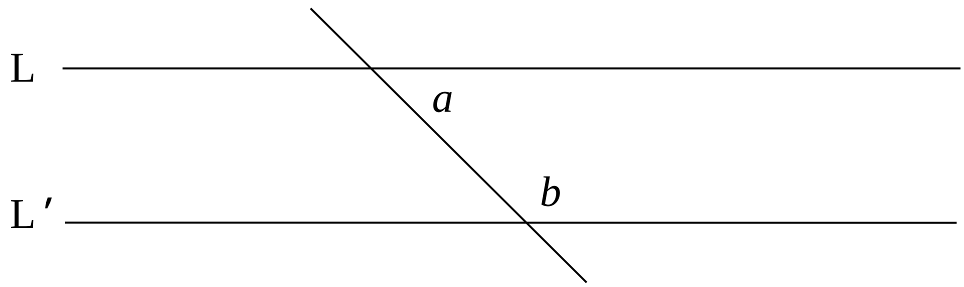
Then we come to the question beyond definition of what is the essential nature of mathematics, and in this there are three well recognized schools. One is known as the *logicists,* of which Bertram Russell is the best known name, who say mathematics is only logic. And the ideal that they have in mind is that by a purely logical process, one that you could program a machine for, you could turn out all mathematics that is and that will ever be discovered in the future. But it hasn't been done, and it runs into difficulties, for you run into some very severe paradoxes; and here's one for you if you can work with it.

Consider the class of all classes which are not members of themselves, is that class a member of itself? Study it, if you don't get dizzy. That question and proposition was sent to a certain Italian—I think it was Peano—who had just finished, I think it was a two volume work on mathematical logic; it was in the press, and that question invalidated his whole thesis. He said it's a hard thing after you've put years into scientific investigation to have the whole tower fall in one moment. You'll find that you can't say yes and you can't say no to that question. Now, there is a paradox in that, that has grown out of it, the very view of the nature of mathematics. I'm wondering if they're not trying to make it a bit too pure; when I speak of it as purity in this sense, it particularly involves the elimination of intuition; reducing it to a logical process that has no more use for intuition—no more need of it.

Now, there's another school, particularly connected with the name of Hilbert, which is called the *formalistic* school. Now, the story here, not connected with logic in the sense that it was the case with Russell, but with a story of the extraordinary forms of geometry.

I'm getting at something here in all of this. To many of you, you won't see the point, but we're learning something about the powers and the limitations of pure thought, and it's precisely in mathematics that we find pure thought; and that's why I think it's worth your while even though you do not have a mathematical background to get some of this—and also be a bit prepared for some possible adversaries.

When Euclid wrote his geometry, he had a number of propositions which he called axioms, and axioms were defined as self-evident truths—something everybody would agree was true—and then from that he started out and tried to deduce all the rest from those. One of those axioms seemed awfully complex, known as the twelfth axiom, known also as the parallel axiom—something like this: if the sum of these two interior angles on the same side of a transversal crossing two lines equal two right angles, then these lines will not meet.



Now, that looks like a theorem, something you had to prove; actually it's an axiom. In your present books—if you study Euclid anymore, I don't know whether you do—you probably find it in this form: through a point in a plane, you can draw one and only one line parallel to a given line. That's the form it takes today. That was the form Euclid left it in. Many mathematicians tried, since it looked like a theorem, tried to prove it from the other axioms, but with complete failure. Then the next step was to see if you could assume something different.

Now, Lobachevsky and Bolyai, independently I believe, assumed that through a given line you could draw through a point two parallel lines; that is, in other words, neither of these lines would touch that line short of infinity—just assumed it. Now, it doesn't look like it; that isn't the point. Can you then logically with that axiom build up a geometry that is self-consistent? The answer is, yes. It was done. Every proposition in Euclid that depends upon this twelfth axiom, so-called, will be different in this geometry. Among other things, you're familiar with the statement that the sum of the interior angles of a triangle are always equal to two right angles. In this geometry, it's always greater than two right angles.

Another man, independently, Riemann—and this is particularly important—assumed that you could not draw any line through a point that would be parallel; that any line would meet the given line in a finite distance; that instead of meeting at infinity, would meet in a finite distance. And in that geometry every line—straight line now—followed in that direction would lead you back into it this way. Now, that isn't according to your intuition, but there's no logical failure in that geometry either. It's self-consistent. Now, in mathematics, according to Russell, any conception exists which is self-consistent; therefore, Riemannian and Lobachevskian geometry exist. About fifty years after Riemann, Einstein came along and developed his General Theory of Relativity, and found that the Riemannian geometry fit his conception of the cosmos. And so Einstein asked, how is it possible that the mathematician thinking in his ivory tower thinks what is true of the universe outside?[6] It's a nice question.

If a pure mathematician—now remember Riemann is a pure mathematician; he's interested in this intellectual exercise. In a way you could say he's doing it for the fun of it. Every pure mathematician does that. A pure mathematician is like the gods, he does nothing because it's his duty; he does it for the fun of it. That's the way the free souls

[6] E. T. Bell, *Men of Mathematics* (New York: Simon & Schuster, New York, 1937), xvii:

> How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?

always are, and there's a good deal of that freedom illustrated in the mathematician. There's no duties for the free soul; he does what he does spontaneously; but what he does spontaneously accomplishes the best results. So the pure mathematician goes into the ivory tower, and he thinks for the fun of it; and then someone else comes along, takes his product, and finds that it gives you a control over some department of nature. And about that time the poor pure mathematician gets sore; his beloved purity is now corrupted by use. There is the story of the mathematician who had an idea that seemed far out, and he said, "I thank God there's no conceivable use of this thing." And now, is that so strange? An artist makes a beautiful picture, and somebody comes along and uses it for selling Coke. How does the artist feel? Well, that's the way the pure mathematician feels. We're not all made over the same pattern. And here's a fact—

Turn it over.

Here's a fact—

I don't know, sure you're not erasing?

—that which is—I forget, well now I lost what I was going to say. That happens. Oh, yes, yes, yes, here's the fact. Practically all of mathematical creation has been done by the pure mathematician sitting in his ivory tower, and it is well known that if you try to produce for a use, you don't produce. The one great exception to that was Sir Isaac Newton's development of the theory of fluxions, which is another name for the calculus. His interest was in a cosmo-conception, and mathematics for him was only a tool. And the same may also be said about the mathematics of Wiener in our own day, one of the chief theoreticians behind the development of the computer, that he did not have the pure mathematical orientation, but an applied one.

Now, let us look at this. Oh, here's a story. This involves what at least in my day was the greatest of the Indian mathematicians, East Indian, Rasmuchin,[7] I think, and Hardy, an English mathematician. They were both of the pure type. Hardy tells, in one of his lectures, of going in a cab to visit this Indian mathematician when he was in England. When he came in he said, the number on my cab was 1729, a very dull number. But the Indian friend said, no on the contrary, it's a very interesting number, for it is the smallest number that is the sum of two cubes in two ways. Now, I'll leave with you the problem of finding those two cubes in two ways. You know, the liberated soul doesn't work, he only plays. His action is a spontaneous expression of delight, but out of that come the greatest creations of all.

I didn't finish up what I was going to say—let me see what our time is; well, a few more minutes if you can stand it—on the development that led to the *formalists*. When this new development on Euclidian geometry took place, this was in effect a revolution in the whole conception of the nature of mathematics. The whole idea of the *axiom* went overboard—of the innate ideas that are natural upon which your logical constructions were made—and in place of them we have substituted what are called only *fundamental assumptions*. Ultimately that led to the formulation of all sorts of

[7] Wolff meant to say "Ramanujan." The answer may be found in Carl Boyer's *A History of Mathematics* (New York: John Wiley & Sons, 1968), 223.

extraordinary geometries, and there's an example of one here. Those of you who know your Euclid may not perhaps recognize this.[8] We have as our fundamental assumptions:

> Axiom 1: If *A* and *B* are distinct elements of *S*, there is at least one *L-class* containing both *A* and *B*.
>
> Axiom 2: If *A* and *B* are distinct elements of *S*, there is not more than one *L-class* containing both *A* and *B*.
>
> Axiom 3: Any two *L-classes* have at least one element of *S* in common.
>
> Axiom 4: There exists at least one *L-class* in *S*.
>
> Axiom 5: Every *L-class* contains at least three elements of *S*.
>
> Axiom 6: All the elements of *S* do not belong to the same *L-class*.
>
> Axiom 7: No *L-class* contains more that three elements of *S*.

Now, here's an application of this geometry.

> Let us suppose there is a banking firm with seven partners. In order to assure themselves of expert information concerning various securities, they decide to form seven committees, each of which will study a special field. They agree, moreover, that each partner will act as chairman of one committee, and that every partner will serve on three, and only three, committees. The following is the schedule of committees and their members, the first member being chairman:

Domestic railroads	Adams	Brown	Smith
Municipal bonds	Brown	Murphy	Ellis
Federal bonds	Murphy	Smith	Jones
South American securities	Smith	Ellis	Gordon
Domestic steel industry	Ellis	Jones	Adams
Continental securities	Jones	Gordon	Brown
Public utilities	Gordon	Adams	Murphy

Shades of Euclid. That's what modern geometry has become. In other words, that combination fits that. Geometry used to mean earth-measurement; no longer is a proper term.

Now, the *formalists* say mathematics is a game, playing with formal entities that don't mean anything but which you play seriously like chess.

[8] Edward Kasner and James Newman, *Mathematics and the Imagination* (New York: Simon and Schuster, 1940), 151-153.

We'll take up next time the *intuitionalists*, a conception of Spengler, and then my own. We haven't gotten as far today as I expected. Shall we rise and say our words?

Let there be peace within the universe.
Let the power of the warriors of light be made manifest.
Let wisdom guide us and love protect us throughout our lives.

Peace be with you.

And with you, peace.

See you tomorrow night.