In what has gone before, I briefly referred to the use of the study of mathematics as a discipline in concentration which would serve to clarify the mind and to eliminate the jumping about of that which is frequently called the monkey mind; that this is a method that can act as a substitute for many of the disciplines that are offered in certain forms of Oriental yoga. But there is more to be said about the office of mathematics in connection with yoga. It is not only a training of the mind in concentration, but it occupies a special position in the Sangsara that is not held by any other discipline, and that I would like to develop at more length.

Just what is mathematics? I will introduce the subject by a few quotations from individuals that have been very prominent in our intellectual history:

A mathematician who is not also something of a poet will never be a complete mathematician.¹

Quoted from Karl Weierstrass, who was one of the greatest analysts of all time, and who produced a system in which the conception of motion was eliminated and in its place introduced the conception that objects occupied points in space and time without motion—different points at different positions in space and time.

Next:

I have heard myself accused of being an opponent, an enemy of mathematics, which no one can value more highly than I, for it accomplishes the very thing whose achievement has been denied me.²

—Johann Wolfgang von Goethe

Number rules the universe.³

—From the Pythagoreans.

Pythagoras was a figure of supreme importance in the whole history of Western thought. He was a contemporary of Buddha, and has been called by Indians, the Foreign Master. He was proficient in three domains. First of all, he was a major mathematician who introduced the supreme principle of proof into mathematical systems. He made

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² Ibid.
³ Ibid.
mathematical discoveries, including one that was far beyond the understanding of the Greek mind at his time, and apparently even beyond his own understanding—the first irrational number, the square root of 2. He was a philosopher and he was a cosmologist who introduced the conception of spherical worlds, and that was quite foreign to the thinking of the time. And, he was in addition, a mystic.

But there is another reason for the high repute of mathematics: it is mathematics that offers the exact natural sciences a certain measure of security which, without mathematics, they could not attain.4

——Albert Einstein.

To create a healthy philosophy you should renounce metaphysics but be a good mathematician.5

——Bertrand Russell. An outstanding contemporary authority on the logic of mathematics.

Mathematics is the only good metaphysics.6

——Lord Kelvin.

I shall return to these two quotations later.

How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?7

——Albert Einstein. This statement will be of particular interest to us later.

Every new body of discovery is mathematical in form, because there is no other guidance we can have.8

——C. G. Darwin.

God ever geometrizes.9

——Plato.

God ever arithmetizes.10

——C.G.J. Jacobi.

And finally:

4 Ibid., xvi.
5 Ibid.
6 Ibid.
7 Ibid.
8 Ibid.
9 Ibid.
10 Ibid.
. . . the Great Architect of the Universe now begins to appear as a pure mathematician.\textsuperscript{11} 
\hspace{1em} — J. H. Jeans in \textit{The Mysterious Universe}.

Clearly these statements from men of proven and well-recognized ability imply that there’s more in mathematics than simply the popular conception which scarcely goes beyond the numerical processes that form the basis of accounting. And, indeed, this is so. We will look into the question of defining just what mathematics is, and we will find that it is not too explicitly determined. If you look up the word in an inadequate dictionary, you’ll find the definition that mathematics is the science of quantity, but this is manifestly very incomplete. Mathematics does indeed include the science of quantity, but goes much further. Generally, quantity is viewed as metrical properties, and there is a vast development of metrical relationships. But it also includes descriptive properties, that is, properties that do not involve at all the notion of quantity. This is exemplified in the discipline known as projective geometry, where all elements—points, lines, and planes—are regarded as unlimited in extent, except that the point represents position alone. All lines are infinite in extent and all planes are infinite in extent. And there are no quantitative relationships considered between these elements, but only descriptive relationships. Yet, these relationships produce effects that are utterly surprising, and there is a very frequent experience of that which is known as mathematical beauty in this field.

Commonly, mathematics is defined as including the two fields of \textit{number} and \textit{space}. This is the sense in which it was discussed by Immanuel Kant in the \textit{Critique of Pure Reason}—most specifically in his “Transcendental Aesthetic.” Kant thought that he had established the basis of mathematics in this way: that he had derived geometry from space and arithmetic, or the whole field of number development, from time. But with respect to the latter, there was a serious challenge by Bertrand Russell, and this explanation as to how pure mathematics is possible by Immanuel Kant may have to be seriously reviewed.

Kant’s division of the discipline of mathematics into the two zones of arithmetic and geometry, we know today was wholly inadequate. To be sure, the vast mass of mathematical literature is concerned with number, and a somewhat smaller mass with geometry, but the various branches of mathematics include more than that. There is, for instance, the discipline known as the theory of groups, which deals with the relationships of operations; and then there is the algebra of logic, which is not concerned with space or number at all, but with a symbolic way of dealing with all logical problems. In fact, Bertrand Russell has said that the founder of this form of mathematics, Mr. Boole, was the first pure mathematician of all.

The definition of mathematics commonly found in an ordinary dictionary is wholly inadequate, for there one may find this statement: that mathematics is the science of quantity, which now it should be seen is far from adequate enough. In the article on mathematics in the 9th edition of the \textit{Encyclopedia Britannica}, there is a statement that gets closer to the heart of the matter. This article was written by

\hspace{1em} \textsuperscript{11} Ibid.
Williamson, a well-known text book writer in the field of calculus, and there he says: “Any conception which is completely determined by a finite number of specifications is a mathematical conception.”\(^\text{12}\) What this implies is that the essence of mathematical formulation is precision in determination. It is the—whenever conceptuality attains high trenchancy and clarity, it becomes mathematical. The emphasis then is upon trenchancy and clarity, not a specific subject matter. Thus we may say that when thought in the conceptual sense dealing with any subject matter whatsoever becomes wholly pure, it becomes mathematical.

Let us return now to two statements made by Bertrand Russell and Lord Kelvin respectively. The first is, “To create a healthy philosophy, you should renounce metaphysics, but be a good mathematician.” And second, “Mathematics is the only good metaphysics.” This introduces us to the problem with which we are specifically concerned, the tie-in between mathematics and metaphysics. Immanuel Kant long ago saw that there was a tie-in between these two. It will be remembered that the Critique of Pure Reason was the result of the work of David Hume, who carried the premises of John Locke to their logical conclusion. The consequence of this was that if we start with the assumption of John Locke, that all knowledge comes from experience, then we would have no knowledge whatever of law, but only the impact on the senses of events as they unfold before us in a meaningless phantasmagoria. This aroused Immanuel Kant, who realized that this meant the annihilation of all possibility of science in every sense—science in the empiric sense and in the mathematical sense—and implied the impossibility of any such thing as metaphysical knowledge. He reestablished the possibility of mathematics and of empiric science by the analysis produced in that work, but I shall not go into that in detail at the present time. But in the “Introduction” he makes the note that David Hume’s analysis destroyed the possibility of pure mathematics just as much as it destroyed the possibility of a pure metaphysics; and, as a matter of fact, mathematics works indubitably and renders possible effects that are obvious to us in the range of our actual experience.

Now, this leads to a question: what is metaphysics? Immanuel Kant was unable to establish its possibility. In other words, the possibility of answering questions of the type: is there a God, is there freedom, and is there such a thing as immortality—and other questions of that sort. Clearly, by the operation of the intellect alone, he proved that we cannot achieve answers to the metaphysical questions, though these are the questions of most profound importance. Bear in mind that however much we can deal with the transitory problems of life here in this world, there remains the mystery imposed by the universal experience of death in the physical sense. Does this mean termination or does it mean a change in the shape of the forms of consciousness in which we move? No question is of more vital importance than this. It is central in the whole field of religious orientation.

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\(^{12}\) *The Encyclopedia Britannica*, vol. 15 (Chicago: The Werner Co., 1894), 629:

Any conception which is definitely and completely determined by means of a finite number of specifications, say by assigning a finite number of elements, is a mathematical conception.
Kant, who was himself a religious man, could give no comfort here. And here I would introduce a definition of metaphysics that falls outside the range of his understanding of this subject matter. It takes this form: metaphysics is the formless knowledge given by nonsensuous immediacy. Ordinarily we think of immediacy as the impact of contents upon consciousness through the action of the senses. This is the impact as it is before interpretation, and in that sense it is well-defined by the words of William James who said it was a “...blooming, buzzing confusion.”¹³ In this sense, it is an impact of an indeterminate somewhat which, by reflection, we have broken up into more or less determinate images; but this involves the cooperation of the conceptual entity. The immediacy is that raw “...blooming, buzzing confusion.” What I am suggesting is another immediacy which is nonsensuous. In other words, the knowledge derived through identity between the knower and the known, or knowledge through identity—a knowledge which also does not have the form which is characteristic of our familiar conceptuality, but has indubitable value of a most extreme importance. Nonsensuous immediacy is attained only through yoga, considered here as a state of consciousness rather than as a methodology, for there is plenty of evidence that such states of consciousness do arise spontaneously, as is revealed in the history of the various religions.

For continuation of the discussion I shall requote my definition of metaphysics. It is as follows: pure metaphysics is the formless knowledge given by nonsensuous immediacy. Along with this, there are two additional statements which I suggest we may entertain as postulates: pure mathematical knowledge gives the principle of order or law governing the cosmic and supercosmic; third, thus mathematics is the path between the cosmic and the supercosmic.

I realize perfectly well that the first two statements are not demonstrable in the discursive sense. They are known, if known at all, by direct insight or imperience,¹⁴ but for him to whom they are not an immediately known principle, it is possible to treat them as postulates for a consideration of the consequences which follow. This is the familiar method of mathematical reasoning. It starts from basic assumptions. These assumptions ordinarily are quite arbitrary, not in any sense proven, thus taken as the ground upon which further reasoning is based. I suggest for him who hears or reads, that he treat these two statements as such postulates and see if the consequent drawn from them does not follow. The consequent here is the statement: thus mathematics is the path between the cosmic and the supercosmic. That is why I call it a way of viewing the Path.

But there are ways of viewing mathematics which are generally current that are incompatible with this interpretation of its possible office. Thus, we have the point of view introduced by Bertrand Russell in which it is maintained that mathematics is logic,

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¹⁴ For the definition of ‘imperience’, see the audio recordings “General Discourse on the Subject of My Philosophy,” part 10, and “On My Philosophy: Extemporaneous Statement.” In speaking of introceptual knowledge, Wolff says, “The third function therefore gives you imperience, not experience. It is akin to sense perception in the sense of being immediate, but is not sensuous.”
and only logic. These are known as the *logicists*. There are those who follow the standpoint of Hilbert, that mathematics is merely formal relationship without meaning. They are known as the *formalists*. And there are those who are called the *intuitionists*, who are represented by men like Kronecker, who said, “God made the integers, all the rest is the work of men.” These are the three current interpretations. A different view is presented by Spengler in his *Decline of the West*, where he says that mathematics takes a form which is representative of the various cultures. Thus, with the classical man, mathematics was conceived of as a study of bodies, whereas, with modern man, in the sense of Western culture, it was conceived of as a study of space.

None of these are adequate for our purpose, and I offer, therefore, this definition: that mathematics is logic plus Vision—with the Vision spelt with a capital ‘V’. In other words, if you take mathematics in the purely formal sense of the formalists, or purely in the logical sense, it is empty. It is form without filling or form without substance. If, however, it is viewed as logic plus Vision, it has filling, and that makes all the difference in the world. But Vision is less than Realization or Enlightenment. It is, as it were, a view of a distant possibility, whereas, Realization, in the fundamental sense, or Enlightenment is the immediate attainment in a substantial sense of the contents that were part of the Vision.

The immediate attainment of the content of the Vision is the metaphysical knowledge with all its immeasurable richness and its saving power. This metaphysical knowledge may be attained without the knowledge of law and order, in which case its substantive richness is realized, but it remains incommunicable. It is described as a formless ocean of knowledge which cannot be carried to others by any means of communication. But with knowledge of the principle of order and law, communication in substantial degree becomes in principle possible. And if others than the individual who himself has attained Liberation are to benefit from this Liberation, communication becomes immensely important.

For the relationship between mathematical and metaphysical knowledge, I suggest a homely figure. Let us take the figure of a nut consisting of a shell and a kernel where the shell represents mathematical knowledge and the kernel represents metaphysical knowledge. So when Bertrand Russell says, “To create a healthy philosophy you should renounce metaphysics but be a good mathematician,” he is saying in effect, crack the nut, swallow the shell, and throw away the kernel. But when Lord Kelvin says, “Mathematics is the only good metaphysics,” he in effect is saying, swallow the whole nut. That is all right, but it might be a bit rugged. But, it might be better to take out the kernel and swallow it, meanwhile, preserving the shell for future reference.

One of the quotations given by Albert Einstein brings us to the very edge of mystery itself, the mystery underlying not only this universe, but the All—both cosmic and supercosmic. It is this quotation, “How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the subjects [objects] of reality?” Back of this remark of Albert Einstein there lies a bit of history that produced a revolutionary effect in our understanding of the nature of

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mathematics. All of you, I assume, have had acquaintance with the geometry that came down from the Greeks which is commonly known as Euclidean geometry, a subject taught in all high schools worthy of the name. You will remember that there are given in that discipline a number of statements which are called axioms, and axioms are defined as self-evident truths, namely, ideas that are not proven but are supposed to be seen as true just immediately, as it were. Among those axioms there is one, namely the twelfth, which is called the parallel axiom. It was stated by Euclid in a form that looked very much like a theorem which could be proven. In our text books it is in the form such as this: that given a line on a plane and a point outside that line, one and only one line can be drawn which is parallel to the given line. But because the form was so much like a theorem, there was an effort to try to prove it from the axioms that preceded, and all this effort failed. Then the effort was made to assume some different form for this axiom. One was to assume that two lines could be drawn that were parallel, and the other was to assume that no line could be drawn that was parallel; in other words, that any line whatsoever through that point would intersect the given line in a finite distance. Two different geometries were thus developed which are now known as non-Euclidean geometries, and they were found not to involve any contradiction at all, that they could be built up consistently and stand on their own feet, yet produce different theorems from those which are to be found in Euclid. It was therefore evident that the parallel axiom was not necessary.

One of these, the one known as the Riemannian geometry which assumed that through that point any straight line would ultimately meet the given line in a finite distance, proved to have special importance and led to special consequences. Ordinarily, when we say that a line that is parallel it is meant that it meets the given line at infinity, but in no finite distance. When it is understood as meeting the given line in a finite distance, it results in producing a finite space and one which is commonly called a curved space. Now, when Einstein stepped from his special theory of relativity to the development of his General Theory of Relativity, he found that the facts of physics satisfied the conditions imposed by the Riemannian geometry, namely, that at the state of our then known knowledge of physical nature, in the broad cosmic sense, could be integrated in this system of Riemann; yet, Riemann did not have in mind any practical application of his system whatsoever. It was a product of pure, detached thought unconnected with physical reality, yet in fitted the reality that appeared at the stage of development of our physical knowledge at the time of Einstein. And Einstein asks, “How is that possible?” And right here is a vast mystery. How is it that the pure thinker, in spite of himself, thinks that which is true of the reality which was not in his mind at all when he began his development of the system? This causes us to recall the music of the spheres of the Pythagoreans if we think of the word music as really referring to a principle of harmony, and also of the pre-established harmony of Leibniz.

To introduce a point which I would like to bring forth in connection with this subject matter, I will relate an incident which happened during my academic years. At the time I was a major in mathematics and had psychology as my principle minor. At the time I also was taking the course in non-Euclidean geometry. There is in this geometry a theorem which runs this way: the sum of the interior angles of the triangle are less than, equal to, or greater than two right angles in the three systems respectively, namely, the systems of Lobachevsky, the Euclidean system, and the Riemannian system. Over at the psychological laboratory, I read this theorem to a graduate student
and he paused, stared awhile, and said, “That is absolute intellectual anarchy.” I might have said in rejoinder, if I had thought of it at the time, “Not so if you have enough mathematics under your belt.” The point is that it all depends on perspective, and is not a matter of intellectual anarchy at all. From the perspective represented by Lobachevsky, the sum of the angles is less than two right angles; from the perspective of Euclid, it is equal to two right angles; and from the perspective presented by the assumptions of Riemann, it is greater than two right angles.

To illustrate how it could be greater than two right angles, for instance, we will consider a representation of a Riemannian surface which can be visualized. We can take the surface of a sphere as replacing our ordinary conception of a plane surface, though he was speaking of plane geometry in his previous representation. Now, on the surface of a sphere we can identify certain points arbitrarily as a pole, and a point opposite the given pole as another pole. Equidistant between these points, we can have a line which we would call, ordinarily, the equator. And, then, we can have meridians of longitude, each of which is perpendicular to the equator, and these meridians meet at the poles. They thus delineate two triangles known as spherical triangles. At the equator, the meridians are perpendicular to the equator, hence, there you have two right angles, and they have an acute angle at the poles. The sum of the three angles in that case is greater than two right angles.

The space of Einstein, the three-dimensional space, is also curved and here we come to a conception that is beyond the capacity of visual imagination, although it is not beyond the capacity of conceptual manipulation through mathematics. Here is one of the realms where conceptual thought transcends the possibility of the image or any imaginal imagination; and this, I may add, is one of the reasons why I rate the power of conceptual thought as transcending all the powers of the image. And this may hint at how it is possible to build a conceptually based yogic method which is effective and which makes no use of the image or of the physical organism at all.

In my academic experience of moving between the mathematical and the psychological departments, I had this realization, that there was a certain hostility in the perspective and operations of the two disciplines, one that was like a kind of hostility or a sort of adversary relationship. It was as though when moving in the domain of mathematical thought one learned to soar into trans-cosmic realms, for mathematics is fully familiar with the infinite and the infinitesimal; and then, when entering the psychological laboratory, it was as though one were punctured and deflated, and forced to crawl on his knees to deal with that which was all too small and too insignificant. They pulled apart, as it were, and it was years later that I found the explanation for the difference. This explanation was found in the first chapter of William James’ [The] Varieties of Religious Experience where he distinguishes between existential judgments and judgments of meaning—the latter he called spiritual judgments. Existential judgments deal with questions of objective fact, circumstance, and the like—the state of consciousness in, uh, or you might say the external associations with states of consciousness, not with the value of the state of consciousness itself. Judgments of meaning, where the orientation is not to the external or biological facts of a given situation, but with the meaningful content of the concepts, leads to value, essentially truth value; whereas, the existential judgment leads to the more or less objective concomitants.
that go along with such states. This gives a clarification. Now, I have no doubt that there are approaches to yoga that will move through the emphasis of the psychological factor, but as I know it, the states realized in the mathematical experience come the closest of anything that can be reached here to the experience that is characteristic of Fundamental Realization—the sense of a soaring in the illimitable; only, in the state of Realization, it is far more complete and integral.

Consider the feeling of wonder and the sense of mystery that surrounds the mathematical experience. I may illustrate this by certain facts which are known about a very famous number, namely, the number which is called “pi.” This is usually viewed as the ratio between the diameter and the circumference of a circle. It is a transcendental number, as we now know. It is a number consisting of one integer, 3, and a decimal that is non-terminating and nonrecurring. In other words, there are an infinite number of decimal points—of decimal integers to the right of the decimal point. But it has other relationships; it is expressed as the sum of certain infinite series and of certain non-terminating fractions. But here is something that is very mysterious: the number \( \pi \) enters into any formula governing the laws of chance. Now, consider what this means. Any situation which we think of as involving random action, or pure chance, involves the number \( \pi \), and that implies that there is no such thing as the authentically random, for even this, which from our outer point of view seems to be pure chance, is governed by law. One example of this is in the following form: arrange a horizontal field, draw lines upon it, say, the length of certain toothpicks, separating the lines from each other. Then throw upon that field a handful of toothpicks. Count the number that do not touch the lines and the number which do touch the lines—put them in separate columns. Make a large number of throws—a thousand or more—and keep the numbers and take the average of them, and they will approach the value of \( \frac{1}{4} \pi \). In other words, though the throws were seemingly utterly uncontrolled by law, they exemplify an underlying law. There is no such thing as pure chance. Law rules all. And the realization of this arouses in one a feeling of awe. This is something that occurs in the rather rare experience of mathematical beauty, where one for a moment has a premonition of that experience which is most completely realized in Fundamental Realization.

Consider the mystery of the relationship in the form: \( e^{\pi i} = -1 \). \( Pi \), as I have said, is a transcendental number; \( e \) is another transcendental number defined as the limiting value of \( (1 + \frac{1}{n})^n \) as \( n \) approaches infinity. It is the base of the Napier or natural system of logarithms. Now, \( i \) is the number \( \sqrt{-1} \), a hypernumber which does not fall in the range of the ordinary numbers with which most people are familiar. The statement is that if \( e \) is raised to the power of \( \pi i \), that is, \( \pi \) multiplied by \( i \), it equals \(-1\). This can be proven. I have gone through the proof. It is in the body of mainstream mathematics—the mathematics which renders the structures, and the machines, and the navigation on sea, air, or in space possible. Yet, the same material proves this relationship. What does it mean metaphysically considered?

What emerges from this discussion is the fact that mathematics is simply pure conceptuality, a conceptuality that is not contaminated with sensuality, or sensuousness; that here is a power that moves on a level freed from the whole of the animal nature; and it is possible for man to function on this level. No doubt, it is a difficult kind of
functioning. No doubt, as we start upon the venture into the mathematical experience, it is difficult to shear off this element of sensuous contamination; but it can be done.

No doubt, the so-called hardheaded engineer and the so-called hardheaded money-maker would scorn this way of thinking, but it is a misnomer to call these men hardheaded, for the pure mathematician is the most rigorous thinker of all. What is really the truth of the matter here is that the so-called hardheaded engineer and money-makers are really hard sensualists or sensationalists, and that is not an ultimately secure base, for any good hypnotist or maya-weaver can destroy the sensational security upon which these men base their operations.

There is an Indian story that illustrates this point. It is said that at one time Krishna, with a disciple, was walking in the desert, and that they came into sight of an oasis. Now, the disciple had asked Krishna as to the theory of maya. What was it like? What produced it? So when they came into the region of the oasis, Krishna said he would rest on a rock and asked the disciple to go to the oasis and get the container they carried filled with water. The chela went to the oasis, found there a farmer with a family, who was raising roses for the purpose of extracting the attar of roses—a perfume. He had conversation with the householder and was invited to take a meal. They ate the meal; they continued in conversation; used up the afternoon; and the chela spent the night with them—meanwhile, forgetting all about Krishna. Then he spent the next day, and presently, day after day, until weeks and months had passed. He finally married a daughter of the householder; and in time, as the years rolled by, the householder died and the chela became heir to the farm. He raised a family, and then there came a great disaster in which the farm and the family and the wife were destroyed, and he found himself wandering forlorn upon the desert. And he came suddenly upon Krishna and said, “You are still here?” And Krishna said, “Only a half hour has passed.” Does sensation provide real security?

I have noticed that psychotherapists tend to identify the sensational order, supposedly held in common, with reality. But what have we here which we certainly know? We have images of various kinds: visual images, auditory images, olfactory images, gustatory images, tactile images, temperature images, and kinesthetic images, among some others. And from these we get the idea of an external object, which, in fact, we project, and then we give to that projection reality. We call this projection the external, real world, but that external real world has no existence for us other than these sensuous images—existences in consciousness. And by orienting ourselves to this supposed external world, we enter into a state of bondage with respect to it, and that causes universal suffering. No doubt, there are pleasurable sensations, but they are only the seductive factors which bind us more securely to that projected external world. When we compare the so-called pleasures of the senses to the experience of a true ananda, they have only the character of a more seductive pain. Sangsara may be likened unto a universal insane asylum, so that the reality of the ordinary psychotherapist is no more than the common ground of universal insanity. All experience of the sensual pleasures involve the morning-after of lesser or major pain and loss. It is not so with the delights of conceptual thought. They produce at least a relatively persistent enjoyment, although even these are less than the true ananda.